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# The Dynamic Analysis of Pneumatic Systems using HyPneu

## Abstract

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Pneumatic systems have been used in industrial applications every bit as long as hydraulic systems. In addition, the performance of a pneumatic system is at least as important as any of the other systems currently in use. However, until recently very little help and few tools were available for the designers of pneumatic systems. The compressibility of the pneumatic medium (air) and the change in volume, pressure, and density as a function of system temperature and pressure have made the dynamic analysis of pneumatic systems very difficult to model and simulate with any degree of success. Like hydraulic systems, the pneumatic counter part is extremely non-linear placing even more of a burden on the designers and would-be simulators. In recent years an approach has been developed and coded into a computer-aided tool called HyPneu which realistically models and simulates both pneumatic and hydraulic systems.

This paper presents a method which is shown to be quick, easy to use, and very accurate in modeling and simulating pneumatic systems. This technique permits the system designer to conduct a dynamic analysis on the system prior to procuring components, assembling the system, and performing testing type of evaluations. This concept will not eliminate system testing completely but will turn such efforts from development activities to verification activities.

## Introduction

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Pneumatic systems are an important part of the modern industrial world. They are many times used where compressed air can readily be obtained and where horsepower requirements are not large. Pneumatic systems are very effective in situations where the motion can be limited by a mechanical stop such as exists in what is often called a bang-bang control system. The development of all fluid power system is very complex due to the nonlinear nature of the expressions which describe the action of the components and systems. Pneumatic systems are even more complicated by the fact that the fluid used in this "fluid power system" is extremely compressible. In hydraulic systems the medium is somewhat compressible as attested to by the bulk modulus. However, the magnitude of compressibility between air and liquid is very large. In addition, when the air is compressed the density changes significantly which even further

complicates the analytical considerations. Due to the complexity of the models which realistically describe fluid power components and systems, designers have elected to use only steady-state calculations in the process of developing pneumatic systems. However, most problems which occur with pneumatic applications become apparent in the dynamic operation and not the steady-state condition.

In order to analyze any system during dynamic operation it is necessary to develop component and system models which are compatible with effective simulation techniques. The problems which plague pneumatic systems usually occur in a few milliseconds and therefore steady-state analysis will not suffice for the development of solutions. However, it is extremely time consuming and costly to design a pneumatic system based upon steady-state analysis, procure all of the components, assemble the components into a system and then discover that the output of the system does not accomplish the intended objective or is highly unstable. In the past designers of fluid power systems were faced with the problem of not having tools which were capable of realistic dynamic analysis of such systems and therefore they were forced to employ the cut-and-try approach. The obvious tool which is currently available and becoming more available by the day is the computer. However, the modern powerful computer is absolutely useless without effective software.

The advent of the personal computer and the development of computer aided design and analysis software have given engineering design a whole new dimension. Today, it is recognized that an effective computer aided design and analysis procedure literally makes available the brains of the experts that were involved in the creation of the software package. With such a package, even those without a great deal of experience and/or background in the design of systems comprised of sophisticated hardware can confidently explore the use of such systems in the implementation of their new machine. In essence, most successful engineered systems involve diverse technical discipline and most designers will not have adequate experience and knowledge in each and every discipline which should be utilized in producing an effective and efficient machine. To assist in this dilemma an innovative modeling technique has been developed for the design and analysis of fluid power systems

This new software package is called "HyPneu" and is based upon a concept which the developers chose to call "Visual Modeling" or "Interfacial Modeling". In this modeling approach, the appropriate analytical expressions for fluid power components along with many mechanical, electronic, logic elements are represented within the computer package an icon. These character instilled icons then become the building blocks to formulate the desired system. That is, the schematic of the desired system is actually developed on a personal computer using the icons available in the HyPneu program. These icons are patterned after the symbols compiled and published in the form of an ISO document. Once the schematic of the system is drawn using the icons, it can be implemented with the system integrating algorithms. This results in a totally effortless analysis and design task.

This paper discusses the visual system modeling approach and reveals the principles upon which it is based. The technique unifies the various disciplines necessary to effectively design modern systems and seamlessly formulates the governing dynamic equations based upon the topographical information presented by the schematic prepared by the designer. The advantages and benefits which result from the use of the HyPneu software will be demonstrated using practical examples related to pneumatic systems.

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## The Visual Modeling Technique (1, 2)

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**D**esign analysis requires that a model of the physical system be developed. The model may be derived analytically as complete mathematical model, empirically as a physical or test based model, or semi-empirically where a mathematical expression is supplemented by test data. However, in any of these cases the model is just a simplification of reality that can be used to approximately describe the characteristics of

a real system. Additionally, no matter how the model is derived, it is likely to be useless unless the model can be converted into a form that is suitable for computer manipulation. It should be noted that modeling is not an addition to the design process because the designer must at least formulate a conceptual model, a schematic model most likely, and/or a mathematical model prior to the final design and manufacturing activities.

At the theoretical development level, modeling is a way of formalizing and documenting the thought process so as to make the characteristics of even the most complicated system tractable. In general, a model can be considered as a set of equations. Mathematically speaking, equations relate some unknown quantity to known quantities, and the values of the unknown quantities can be determined in terms of the known entities using the equations. They can also be manipulated so that what was formerly an unknown quantity now becomes a known quantity and vice-versa. In other words, a model is used to transform a set of known quantities (inputs) to obtain a set of entities (outputs) that were originally unknown. The repetition of this input/output transformation process with the consideration of the time effect characterizes the dynamic performance of a physical system. In this regard, the objective of modeling is to derive a set of equations that can be used to portray the behavior of the system according to a set of known quantities.

The behavior of a real physical system is controlled by the flow, storage, and interchange of various forms of energy. Therefore, system performance can be explored analytically if the energy actions and interactions in a given system can be described mathematically. Depending upon the manner in which the system elements handle the flow of power and energy transfer, system elements can be classified as either energy storage or energy dissipation elements. Energy storage elements can be further separated into the type of parameters that are critical -- across type (A-type) parameters or through type (T-type) parameters. The A-type variables act across the element. Examples of A-type variables are pressure, velocity, and voltage. The T-type parameters pass energy through the elements such as flow, force, and current. The energy dissipation elements are normally called D-type elements. They are characterized by orifices, friction, and resistors. Since all system elements can always be represented as A-type, T-type, or D-Type regardless of the engineering discipline (hydraulic, mechanical, electronic, etc) involved, all engineering system models can be unified using these principles. This unique parameter unification approach forms the basis by which the computer can process the interfacing between the different engineering models and provides the possibility for visual modeling.

A system is a composition of many interacting components. The interaction takes place by the exchange of energy between the components. System modeling handles this interaction characteristic between the various components of a system. Unlike component modeling, which is subjective in that it deals only with the component, system modeling is more objective in that it considers the synergistic effects among components as well as their interaction with the surrounding environment. For the convenience of the modeling of a system, it is more appropriate to consider a component to have a set of well defined power ports through which all energy transfers are accomplished. Since all physical systems obey the law of conservation of energy, this fact can be used to establish a relationship between the energy transactions at the various ports of the system. In order to account for the dynamic effects, it is a common practice to use the rate of energy transfer (power) at each component port. The use of power variables has a great advantage in system modeling due to the fact that the power at each port is the product of two dynamically meaningful quantities -- the A-type parameter and the T-type variable. For example, hydraulic power is a function of pressure and flow, translational mechanical power is a function of velocity and force while voltage and current provide electrical power. Consequently, the A-type and T-type energy variables are the communication language among components. This power port approach dramatically simplifies the component model structure because internally the only variable required for the formulation of the component model is the D-type variable. This permits the component to be represented by an appropriate icon with its mathematical model appended.

Visual modeling consists essentially of both component and system modeling. At the component modeling level, this approach uses an icon for each engineering component of interest by setting the A-type variable as the state variable and the T-type variable as the derived parameter. The D-type variables and the model equations are used to derive the T-type variables from the related A-type variables at each power port. For example, a hydraulic orifice has two ports. The orifice flow (T-type) at each port can be calculated from the orifice coefficients (D-type) and the square root of the pressure drop (A-type) across the orifice. At this point, an icon representing the component can be established and includes this A-D-T model structure to form the basic building blocks for the system.

At the system modeling level, when a system is completely and correctly connected, the visual modeling technique not only extracts the A-D-T equations from the component icon, but also employs three very important conditions to form the system model. These three conditions are called compatibility, continuity, and constraints as described below:

**Compatibility Condition** -- This is sometimes referred to as the path law and results in a relationship among the various across variables. Stated very simply, the compatibility condition provides that the summation of the A-type parameters around any closed path of the system must equal zero. On the other hand, compatibility requires equal (compatible) A-type quantities at the points where the power ports are connected.

**Continuity Condition** -- Sometimes referred to as the vertex law, the continuity condition results in a relation among the various T-type parameters. Stated in very simple terms, the sum of the T-type variables at any node (point of port connection) must be zero in the steady state case. There are as many node equations as there are nodes in the system. By considering capacitance effect, these node equations form the system dynamic model.

**Constraint Condition** -- This condition deals with the geometric constraints and the physical boundary conditions of the system. It is the constraints that are imposed upon the motions of the various elements of the system. Normally, the constraint condition is a combination of a set of linear algebraic equations and some known boundary values. In the mechanical system, if the displacement is assumed as an A-type parameter, then the constraint condition and the compatibility condition are equivalent. However, for the convenience of analysis, the visual modeling approach treats the compatibility and the constraint condition separately.

It should be obvious from the preceding statements that in the visual modeling technique the component interaction is handled at the system modeling level and the power variables are unified. This permits the visual modeling algorithm to be implemented on the computer and to be manipulated using any mixed engineering systems. Because a system model is represented on paper visually by using component icons which carry the appropriate mathematical model which describes the performance of each component, this technique is called “Visual Modeling” or interfacial modeling. In the concept of visual or interfacial modeling, the modeling procedures stated above are combined with system circuit drawing routines so that the component icons can be accessed to form system models. This combination has resulted in a complete software package called “HyPneu”. HyPneu is described more completely in the next section of this paper. In addition, the visual modeling technique employed in the HyPneu program is demonstrated in this paper with two pneumatic systems—one fairly simple system which was probably encountered in engineering school and one more complex system generally representing the air suspension systems which can be found on some automobiles and trucks.

## Dynamic Modeling of Pneumatic Systems (3, 4, 5)

**B**asic understanding of the fundamental laws and equations governing the flow of fluids is essential to analytically describe the dynamic characteristics of any pneumatic system. Theoretically, the motion of a given fluid particle in chemical and physical equilibrium can be completely defined if one knows the pressure, temperature, and velocity as a function of space and time. In addition, the fluid properties of density and viscosity are key variables in the determination of the state of a fluid particle. Therefore, for general application, there are seven variables required to describe fluid dynamics. They are: three velocity components ( $u,v,w$ ) of coordinates ( $x,y,z$ ), the pressure ( $P$ ), the temperature ( $T$ ), the density ( $\rho$ ) and the viscosity ( $\mu$ ). Obviously, seven equations are required in order to solve for them simultaneously and obtain the seven unknown variables ( $u,v,w,P,\rho,\mu,T$ ) as a function of time. This is the theoretical basis for the dynamic modeling of any fluid systems including incompressible (hydraulic) and compressible (pneumatic) fluid systems.

The fundamentals of fluid mechanics state that regardless of the nature of the flow, all flow situations are subjected to the following relationships:

- Newton's laws of motion must hold for every fluid particle at every instant. This is the law of conservation of momentum which is normally described mathematically by the well known Navier-Stokes equations. (3 equations)
- The continuity relationship or the law of conservation of mass must apply (1 equation)
- The first and second laws of thermodynamics or the law of conservation of energy must hold (1 equation)
- The equation of state which relates the fluid density to the pressure and temperature must apply. (1 equation)
- The empirical viscosity equation as a function of pressure and temperature must hold. (1 equation)

Obviously, the relationships stated above will form a system of seven equations which are theoretically sufficient to solve for the seven variables ( $u,v,w,P,\rho,\mu,T$ ) of interest. Unfortunately, these equations are highly nonlinear and, consequently, are seldom used in their complete form to set up a fluid system model. In practice, some restrictions are normally imposed on to the flow and operating conditions in order to simplify the dynamic equations.

For the sake of simplicity while not lose the generality, this paper assumes that the gas used in the modeling process behaves as a thermally perfect gas (i.e., the gas obeys the equation of state) and also as a calorically perfect gas (i.e., the specific heats are constant). In addition, assumption is made that the gas inertial effect is negligible, the viscosity is constant and the flow is one-dimensional. Thus, there are four variables ( $u,P,T,\rho$ ) which remained to describe the one-dimensional gas flow dynamics. The corresponding four equations are the Navier-Stokes equation related to the velocity ( $u$ ) in the  $x$ -direction, the continuity equation, the energy conservation law and the equation of the state. In pneumatic applications, flow rate  $Q$ , is normally used instead of flow velocity,  $u$ . Moreover, due to the fact that gas is highly compressible and the mass is constant, the mass or weight flow rate is normally used rather than volumetric flow rate to represent flow capacity. Mathematically, the weight flow rate is as follow:

$$Q_w = \gamma \cdot u \cdot A \quad (1)$$

where  $\gamma$  the local weight density of the gas ( $=\rho g$ )  
 $\rho$  the local mass density of the gas

g the acceleration of gravity  
 A cross sectional area of flow passage

Therefore, rearranging the four governing equations provides the following dynamic equation relating the gas pressure and flow for a control volume.

$$\frac{dP}{dt} = \frac{1}{C_n} [Q_{w,in} - Q_{w,out}] \quad (2)$$

where P the local pressure inside the control volume  
 $C_n$  gas capacitance  
 $Q_{w,in}$  the weight flow into the control volume  
 $Q_{w,out}$  the weight flow out of the control volume

Note that the gas capacitance,  $C_n$ , depends on the compression process. To determine  $C_n$ , the energy equation must be applied to the control volume. In general, it is very difficult to evaluate the  $C_n$  either analytically or experimentally during the compression process. In practice, the  $C_n$  value is evaluated at two possible extreme conditions--isothermal (for a very slow change of P) or isentropic (for a very fast change of P) as shown below:

$$C_n = \frac{g \cdot V}{R \cdot T} \text{ for isothermal process} \quad (3)$$

$$C_n = \frac{g \cdot V}{\kappa \cdot R \cdot T} \text{ for isentropic process} \quad (4)$$

where V control volume  
 $\kappa$  specific heat ratio (=1.4 for air)  
 R gas constant  
 T gas temperature

Note that in Eq.(2), the A-type variable is the pressure, P, and the T-type variable is the weight flow rate,  $Q_w$ . In order to satisfy the A-D-T model format, we need to identify the D-type variable. Undoubtedly, the D-type variable is the flow resistance factor. For gas flow through a sharp-edged orifice, the D that function relates the A-type variable (P) and the T-type variable ( $Q_w$ ) is as follows:

If the flow is subsonic ( $P_{du} > P_{du,critical}$ ):

$$Q_w = C_d A_0 g \sqrt{\frac{2\kappa}{R(\kappa-1)}} \frac{P_u}{\sqrt{T_u}} \left(\frac{P_d}{P_u}\right)^{\frac{1}{\kappa}} \sqrt{1 - \left(\frac{P_d}{P_u}\right)^{\frac{\kappa-1}{\kappa}}} \quad (5)$$

else, if the flow is sonic or supersonic ( $P_{du} \leq P_{du,critical}$ ):

$$Q_w = C_d A_0 g \sqrt{\frac{\kappa}{R \left(\frac{\kappa+1}{2}\right)^{\frac{\kappa-1}{\kappa}}}} \frac{P_u}{\sqrt{T_u}} \quad (6)$$

$$P_{du,critical} = \left(\frac{P_d}{P_u}\right)_{critical} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} \quad (7)$$

where  $Q_w$  weight flow rate  
 $C_d$  discharge coefficient of the orifice

- $A_0$  orifice area
- $P_u$  upstream stagnation pressure
- $T_u$  upstream stagnation temperature
- $P_d$  downstream pressure
- $P_{du}$  downstream to upstream pressure ratio

As can be seen, Eqs.(5), (6) and (7) are quite complicate and inconvenient to use. Consequently, in practice, a simpler parameter called  $C_v$  is more commonly used to represent the nominal flow capacity of pneumatic valves or restrictors. By definition, an opening or restriction has a flow coefficient  $C_v$  of 1.0 if it will pass 1gpm of water at 1.0 psi pressure drop. There are several methods used to obtain the  $C_v$ . The most commonly used one is the NFPA (National Fluid Power Association) equation as follows:

$$C_v = \frac{Q_{ws}}{K_g} \sqrt{\frac{G \cdot T_u}{\Delta P \cdot (P_d + P_a)}} \quad (8)$$

- where  $Q_{ws}$  gas flow in SCFM (at 14.7 psia, 68°F or  $\text{dm}^3 / \text{s}$  at 1 bar, 20°C)
- $K_g$  units conversion factor, 22.48 (US) or 114.5 (SI)
- $\Delta P$  pressure drop ( $P_u - P_d$ ), psig (bar)
- $G$  specific gravity of the flowing medium, (=1 for air)
- $P_a$  atmospheric (absolute) pressure, 14.7 psia or 1 bar

## Example Systems

### Example System No. 1 Air Tank Discharge Dynamics

This example system, as shown schematically in Fig. 1, consists of an air tank identified with reference number N3 and icon ID SP9130 which is pre-charged to 34 psig and plugged by the component with reference number N5. Downstream of the pre-charged tank is a two way two position valve with reference number N6 and icon ID SP 6220 which becomes a fixed orifice when it is completely open. Since this valve is not a modulating valve it will only assume one of two positions -- completely closed w/o leakage or completely open. Downstream of the valve is a second tank with reference number N4 and the same icon ID as the pre-charged tank. This tank has no pre-charge and represents the capacitance of the system. A sharp edged fixed orifice is located downstream of the second tank which is connected to a zero pressure, infinite capacity reservoir (the environment). As was mentioned previously tank 1 is pre-charged to a pressure of 34 psig. In order to simulate the two way valve is opened and tank 1 is permitted to discharge. In using the HyPneu program any nine parameters can be selected for later reporting. In the example illustrated here only three variables were monitored. These three parameters were as follows:

- Pressure at Port 2 of Tank 1
- Pressure at Port 2 of Tank 2
- Flow from Port 2 of the Orifice

The simulation commenced when the two valves were opened at time equal zero. A graph of the simulation results for the outlet pressure of tank 1 and 2 versus time are shown in Fig. 2. As can be seen on these curves the pressure at the outlet of tank 1 slowly decreases while the pressure at the outlet of tank 2 increased at first and slowly decrease along with the outlet pressure of tank 1. In Fig. 3 the outlet pressure of tank 2 and the flow out of port 2 of the orifice is plotted versus time. From this graph, it can be observed that the flow through the orifice increases to a value of about 0.48 SCFM before dropping steadily to zero flow at approximately 17 seconds

Figure 1.  
Schematic for  
Example System No. 1

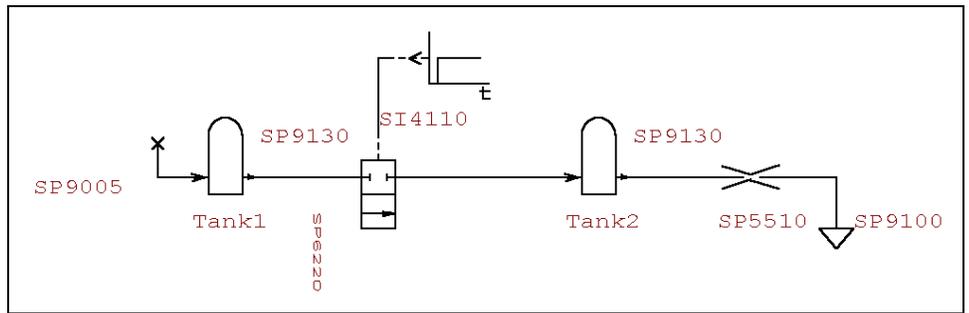


Figure 2.  
Graph of Simulation Results of  
Outlet Pressure of Tank 1 and 2

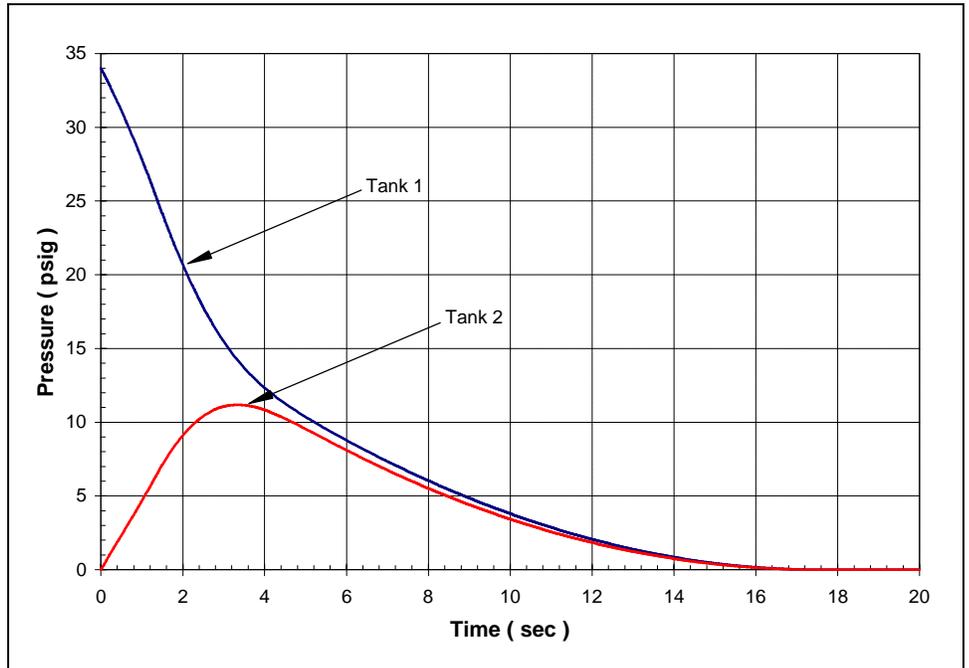
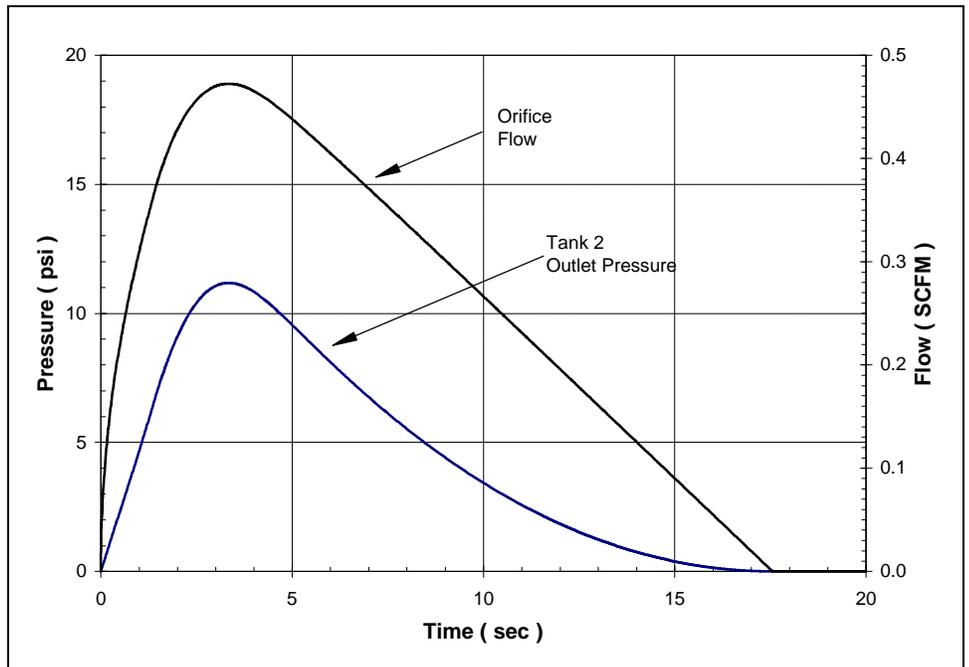


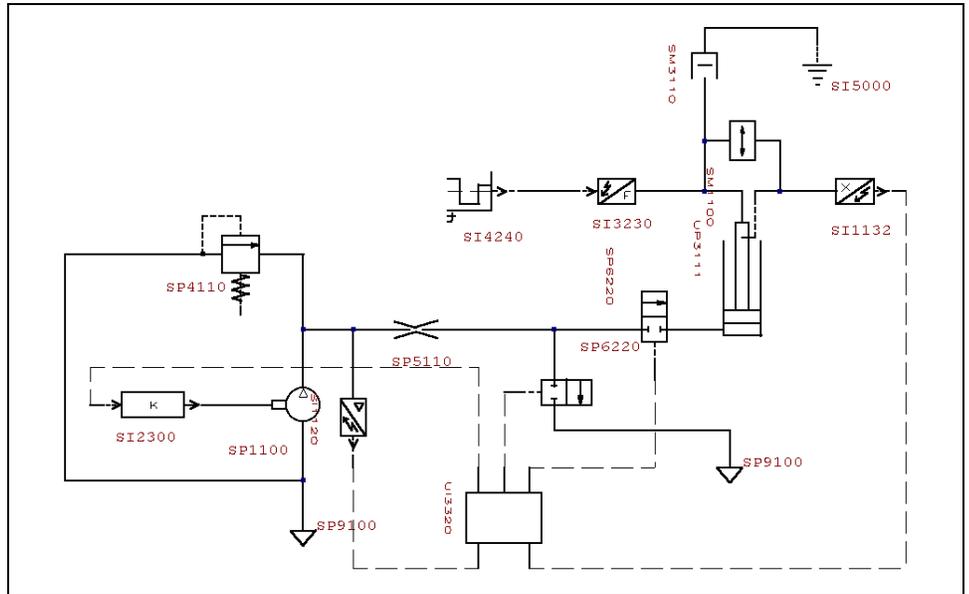
Figure 3.  
Graph of Simulation Result  
of Tank 2 outlet pressure  
and Orifice Flow



## Example System No. 2 Air Suspension System

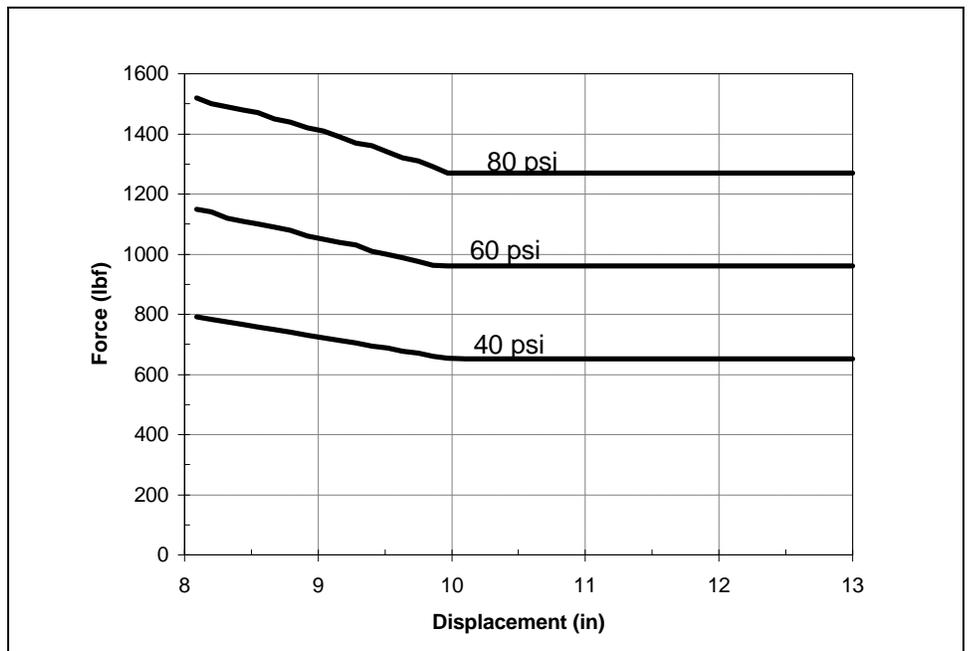
The schematic developed with the HyPneu software is shown in Fig. 4. This system typifies those pneumatic system that are used to control the position of a truck or automobile as the load increases due to cargo. Fig. 4 shows that an air cylinder commonly called as air spring because the housing is a non-rigid structure is attached to each wheel. A feedback pneumatic system supplies air to the air spring to support a larger load at the same height or release air from the air spring when the load is reduced. The duty of this air suspension system is to maintain the height of the vehicle regardless of the load applied.

Figure 4.  
HyPneu Schematic of a typical  
Air Suspension System



The actual air spring is a very unique component and it is represented by a model or icon ID of UP3111 as shown in Fig. 4. This ID number with a prefix U indicates that the model and icon is a special, usually supplied by the user and not a part of the standard component library provided by the HyPneu program. Fig. 5 illustrates the performance of the air spring used in this system.

Figure 5.  
Air Cylinder Model ID UP3111  
Performance Characteristics

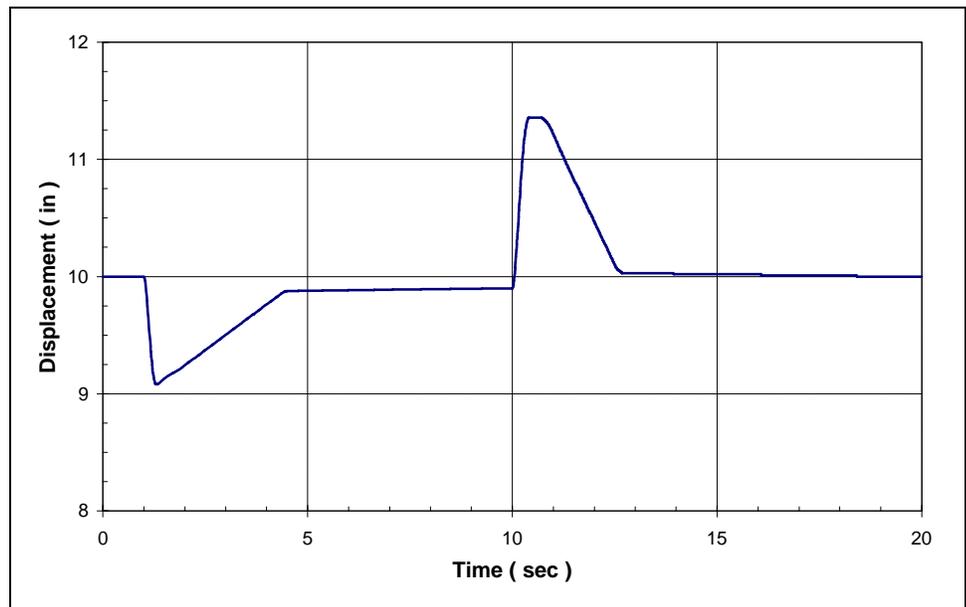


This system was simulated by suddenly increasing the load on the system and then later decreasing the load. The steady state load on the air spring is represented by model ID SM1100 which is set at 1000 pounds in the vertical direction. The change in the load on the air spring is represented by a vertical axial force shown by model ID SI4240 and transformer SI3230. The force is generated by first providing a load signal which comes from SI4240 then converting this force signal to a force vector with an appropriate direction. In this case the load was increased by 250 pounds to a total load of 1250 pounds at one second after the start of the simulation and the load was then decreased to 1000 pounds at a simulation time of 10 seconds. The 1000 pounds was maintained then until the simulation time was 20 seconds. During the simulation, three parameters were monitored as given below:

- Air Cylinder Displacement
- Air Cylinder Inlet Pressure
- Air Cylinder Inlet Flow

A plot of the air cylinder displacement versus time is shown in Fig. 6. As can be seen from this graph, the displacement of the air spring went from the set point of 10 inches to about 9.1 inches when the load was increased from 1000 pounds to 1250 pounds. Then the control system slowly pumped air into the air cylinder to bring the displacement back to the set point. It should be pointed out that there is a dead band in the control algorithm between 9.9 inches and 10.1 inches. Therefore, when the cylinder displacement is in the dead band (tolerance zone) area the control system is satisfied and no further motion will occur. When the load was reduced back to 1000 pounds at the 10 second point in the simulation the air cylinder displacement increased to approximately 11.25 inches and the control system brought it back to the set point in about 2 seconds. A graph of the flow into and out of the air cylinder and the pressure at the inlet port is shown in Fig. 7. When the load was increased to 1250 pounds, the cylinder pressure went from about 65 psig to 82 psig when steady state was again reached and returned to about 65 psig after the load was reduced back to 1000 pounds. The air flow went out of the air spring when the additional 250 pounds of load was placed on the cylinder.

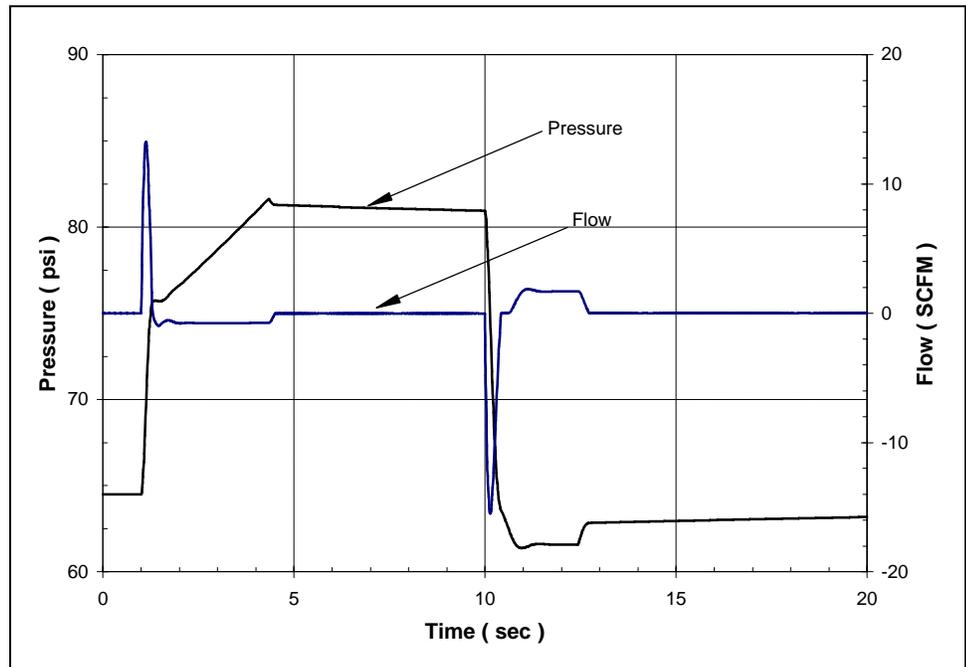
Figure 6.  
Simulation Results of  
Air Cylinder Displacement



The convention with the HyPneu software is that when flow leaves the port it is a positive value and when flow enters a port it is negative. Therefore, when the load was increased on the air spring, the flow spiked to about 12 SCFM out of the cylinder as the displacement decreased. Then the flow assumed a value of approximately -0.6 SCFM while the displacement was returning to the 10 inch set point value. This air flow value was established by the capability of the air compressor used to produce air flow under

pressure. When the load was again decreased the air spring pressure was too high for the new load and the air cylinder extended which produced an air flow into the cylinder of about - 18 SCFM. The air flow went to approximately +1.6 SCFM until the cylinder returned to the 10 inch set point displacement.

Figure 7.  
Simulation Result of Pressure  
and Flow at Air Spring Inlet



## Conclusions

In order to produce a design of any engineered system in an efficient and effective manner it is important to perform as much analysis as necessary before parts are ordered. In the past only steady state analysis was performed because it was extremely difficult and time consuming to perform dynamic analysis. Therefore, when the system did not perform as expected the problem normally fell the dynamic operation. Dynamic analysis requires the development of component models initially and a system model. Then a simulation tool (computer) was necessary to obtain data from the models. Pneumatic components perform in a highly non-linear manner and the energy transfer medium (air) is highly compressible. Both of these facts complicated the modeling and simulation of pneumatic systems.

The visual modeling concept used in the HyPneu program clearly provided the mechanism which forms the necessary unification. This paper has offered two pneumatic systems as examples of the use of HyPneu on these systems. The examples clearly show the potential of this approach in the design analysis of pneumatic systems.

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