

Relief Valve Model

The pressure and flow characteristic curve of a relief valve, such as the one shown in Fig. 6-8, can be described mathematically. Figure 6-15 illustrates a pressure-relief system using a direct acting relief valve. The pressure to be controlled, that is the system pressure, P_s , is sensed on the spool-end area and opposes a spring force setting. The difference between the pressure force and the spring force is used to actuate the spool that regulates the flow to maintain pressure at a spring set value. When the system pressure increases to the preset value, normally called the cracking pressure, the spool moves and opens the control orifice. The resulting increase of flow through the valve relieves the system pressure. The cracking pressure is represented as

$$P_c = \frac{F_{s0} \pm F_r}{A_s} \quad (6-1)$$

where

- P_c = cracking pressure
- F_{s0} = spring force preload ($= k_s \cdot x_0$)
- k_s = spring rate
- x_0 = precompressed spring length
- F_r = Coulomb friction force
- A_s = spool area normal to pressure

Note that the Coulomb friction force acts in a direction opposite to that of spool motion. This is one of the factors that contributes to the hysteresis exhibited by the valve during its opening and reseating as shown in Fig. 6-7.

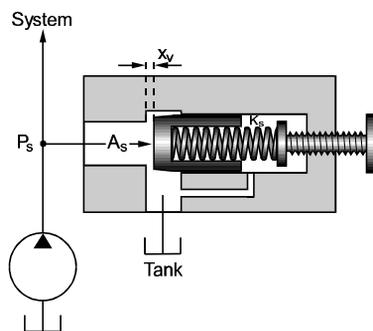


Figure 6-15. A Direct Acting Relief Valve in Activated Mode.

In practice, system relief pressure will be higher than the preset cracking pressure due to both the mechanical and hydraulic spring forces existing when relief flow is passing through the valve. This is normally called a pressure override. The force balance equation which describes the relief pressure as the relief valve is as follows:

$$P_s A_s = k_s (x_0 + x_v) + F_{sf} \pm F_r \quad (6-2)$$

where x_v = valve opening width (spool displacement)
 F_{sf} = steady state flow force ($= C_f P_s x_v$, assuming discharge port pressure=0)
 C_f = flow force area gradient ($= 2\pi D C_d C_v \cos\phi$ for a spool valve)
 D = spool diameter
 C_d = flow discharge coefficient
 C_v = flow velocity coefficient
 ϕ = flow jet angle

Substituting Eq. (6-1) into Eq. (6-2) gives:

$$P_s = \left(P_c + \frac{k_s x_v}{A_s} \right) \left(\frac{A_s}{A_s - C_f x_v} \right) \quad (6-3)$$

In general, a good relief valve is designed to maintain P_s as close to P_c as possible within the full operational range. That is, the relief pressure should increase very little as the flow through the valve increases. This can be accomplished, according to Eq. (6-3), if the flow force factor, $C_f x_v$, is negligible and

$$\frac{k_s x_v}{A_s} \approx 0 \quad (6-4)$$

Equation (6-4) indicates that three design parameters (k_s , x_v , A_s) can be manipulated to minimize the relief pressure increase. This can be accomplished by having a small spring rate (k_s), a small valve displacement (x_v) and/or a large valve area (A_s). Nevertheless, in a direct acting relief valve, such a design approach requires a large valve structure. Thus, in practice, the pilot operated two-stage relief valve is used if a large pressure override is of concern. The two-stage valve normally has a stiff spring in the pilot stage to provide a high cracking pressure. However, it is necessary for the main stage to have a light spring with a large spool or poppet. Such an arrangement will satisfy Eq. (6-4).

Assuming that the flow force is negligible, the equation for the relief valve flow can be written as follows:

$$Q_r = C_d \pi D x_v \sqrt{\frac{2}{\rho} P_s} \quad (6-5)$$

Solving for x_v from Eq. (6-5), and substituting the result into Eq. (6-3) yields the following:

$$P_s = P_c + \frac{k_s}{A_s} \frac{Q_r}{C_d \pi D \sqrt{\frac{2}{\rho} P_s}} \quad (6-6)$$

where Q_r = flow through the relief valve
 ρ = fluid density

Equation (6-6) describes the pressure override characteristic of a relief valve. Note that Eq. (6-6) is a non-linear equation in terms of P_s and Q_r . It requires an iteration process to solve P_s with respect to a given Q_r to obtain the pressure-flow curve for a relief valve.

In practice, the pressure override characteristics are frequently approximated by a linear function. To completely define the relief valve model, let us redraw Fig. 6-8 using zero leakage flow as shown in Fig. 6-16. The characteristic curve indicates that the valve will permit zero flow until the system pressure reaches the cracking pressure. Then the flow area of the relief valve will increase until the system pressure increases by ΔP_r . At this point, system pressure is equal to the maximum pressure, P_m , in the linear range. The valve is now fully opened with a discharge flow of Q_m . The discharge flow will then follow the orifice flow equation when the system pressure is higher than P_m . Mathematically, the model is described as below:

$$\begin{aligned} Q_r &= 0 && \text{if } P_1 \leq P_c \\ &= Q_m \left(\frac{P_1 - P_c}{P_m - P_c} \right) && \text{if } P_c < P_1 \leq P_m \\ &= K_v \sqrt{P_1 - P_2} && \text{if } P_1 > P_m \end{aligned} \quad (6-7)$$

where K_v = valve pressure-flow coefficient $(= \frac{Q_m}{\sqrt{P_m}})$

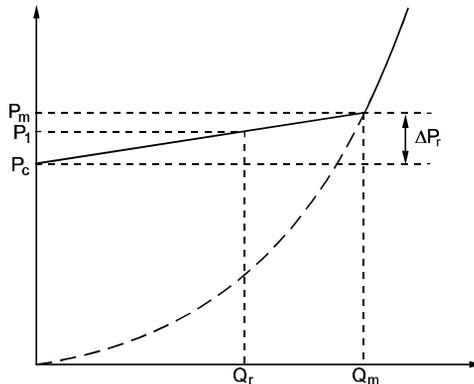


Figure 6-16. Pressure-flow Curve of a Relief Valve.

For a remotely controlled relief valve (see Fig. 6-17), the valve opening is controlled by the remote pressure, P_3 . The flow through the valve will follow the orifice equation. The flow model will then be as follow:

$$\begin{aligned}
 Q_r &= 0 \quad \text{if } P_3 \leq P_c \\
 &= K_v \left(\frac{P_3 - P_c}{P_m - P_c} \right) \sqrt{P_1 - P_2} \quad \text{if } P_c < P_3 \leq P_m \\
 &= K_v \sqrt{P_1 - P_2} \quad \text{if } P_3 > P_m
 \end{aligned} \tag{6-8}$$

Notice that for a remotely controlled valve, P_3 determines the valve opening while $(P_1 - P_2)$ defines the discharge flow rate.

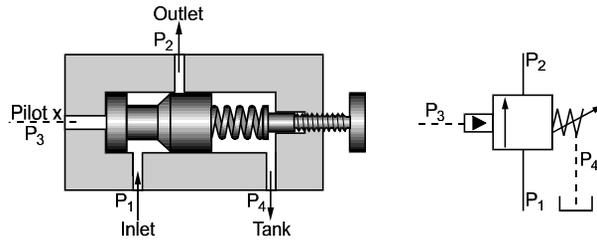


Figure 6-17. A Remotely Controlled Relief Valve with Symbol.

The dynamic model can be obtained by simply adding the inertial force and viscous damping force into the force balance equation, Eq. (6-2), and including an equation relating the rate of pressure change to the net dynamic flows as follows:

$$P_s A_s = k_s (x_0 + x_v) + F_{sf} \pm F_r + m \ddot{x}_v + b \dot{x}_v \tag{6-9}$$

$$\dot{P}_s = \frac{\beta_e}{V_c} (Q_p - Q_r - A_s \dot{x}_v) \tag{6-10}$$

where

- m = spool mass plus 1/3 mass of spring
- b = viscous damping coefficient
- β_e = effective bulk modulus
- V_c = pressure sensing chamber volume
- Q_p = inlet flow rate

Both the static model, Eq. (6-6), and the dynamic model, Eqs. (6-9) and (6-10), are generally solved using a computer.

Example 6-1

Figure 6-18 shows a poppet type relief valve with a half angle of 30° . The pressure sensing port is $0.305''$ (0.775 cm) in diameter. The poppet is held by a spring that has a spring constant of 600 $\text{lb}\cdot\text{f}\cdot\text{in}^{-1}$ (1.05076×10^3 $\text{N}\cdot\text{cm}^{-1}$). The spring is initially compressed $0.122''$ (0.3099 cm). The fluid is MIL-H-5606 which has a density of 7.95×10^{-5} $\text{lb}\cdot\text{f}\cdot\text{s}^2\cdot\text{in}^{-4}$ (845 $\text{kg}\cdot\text{m}^{-3}$). Assume the orifice discharge coefficient C_d is 0.61 . Determine the following:

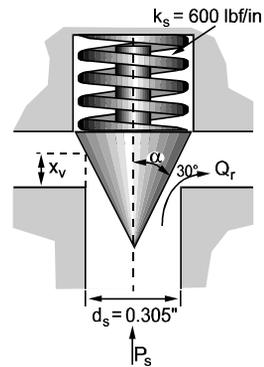


Figure 6-18. Poppet Valve Parameters.

- The cracking pressure if the friction is negligible
- The relief valve flow if the system pressure is 1200 psi (82.74 bar)
- The system pressure if the relief flow is 5 gpm (18.93 ℓpm).

Solution:

- The cracking pressure, P_c , can be calculated by Eq. (6-1):

$$P_c = \frac{F_{s0}}{A_s} = \frac{k_s x_0}{0.25\pi d_s^2} = \frac{(600 \cdot \text{lb}\cdot\text{f}\cdot\text{in}^{-1})(0.122 \text{ in})}{0.25\pi(0.305 \text{ in})^2} = 1002 \text{ psi}$$

In SI units

$$P_c = \frac{(1.05076 \times 10^3 \cdot \text{N}\cdot\text{cm}^{-1})(0.3099 \text{ cm})}{0.25\pi(0.7747 \text{ cm})^2} = 69.078 \text{ bar}$$

- To find the flow through the valve at 1200 psi, we need to know the flow area. For a poppet valve the flow area, A_v , is described by the following expression:

$$\begin{aligned} A_v &= \pi x_v \sin(\alpha)(d_s - x_v \sin(\alpha) \cos(\alpha)) \\ &\approx \pi d_s \sin(\alpha) x_v \text{ if } x_v \text{ is small} \end{aligned}$$

The pressure-flow characteristic of a spool type relief valve is described by Eq. (6-6). For a poppet valve with a small valve displacement, Eq. (6-6) can be rewritten by simply replacing the spool area gradient, πD , with $\pi d_s \sin(\alpha)$ for a

poppet. Therefore, the discharge flow through the relief valve at 1200 psi can be found as follows:

$$\begin{aligned}
 Q_r &= \frac{A_s}{k_s} \left(C_d \pi d_s \sin(\alpha) \sqrt{\frac{2}{\rho} P_s} \right) (P_s - P_c) \\
 &= \frac{0.25\pi(0.305 \text{ in})^2}{600 \cdot \text{lbf} \cdot \text{in}^{-1}} \\
 &\quad \left(\frac{0.61\pi(0.305 \text{ in}) \sin(30^\circ)}{(1200 \text{ psi} - 1002 \text{ psi})} \sqrt{\frac{2}{7.9 \times 10^{-5} \frac{\text{lbf} \cdot \text{s}^2}{\text{in}^4}} (1200 \text{ psi})} \right) \bullet \\
 &= 10.1 \text{ gpm}
 \end{aligned}$$

In SI units

$$\begin{aligned}
 Q_r &= \frac{0.25\pi(0.7747 \cdot \text{cm})^2}{1.05076 \times 10^3 \cdot \text{N} \cdot \text{cm}^{-1}} \\
 &\quad \left(\frac{0.61\pi(0.7747 \text{ cm}) \sin(30^\circ)}{(82.74 \text{ bar} - 69.1 \text{ bar})} \sqrt{\frac{2}{845 \cdot \text{kg} \cdot \text{m}^{-3}} (82.74 \text{ bar})} \right) \bullet \\
 &= 38.13 \cdot \ell\text{pm}
 \end{aligned}$$

- c. It is more complicated to find the relief pressure by a given flow than to find the relief valve flow for a given pressure, because the flow, $Q_r(P_s)$ is non-linear in terms of the pressure. Normally, the solution to a non-linear equation can be approached by iteration with an initial guess. From statements **a** and **b** described in this example, we noticed that Q_r is zero at $P_s = 1002$ psi (cracking pressure), while Q_r is around 10 gpm at 1200 psi. Thus, intuitively, a good first guess of P_s at $Q_r = 5$ gpm would be 1100 psi. Now substituting $P_s = 1100$ psi into

$$Q_r = \frac{A_s}{k_s} \left(C_d \pi d_s \sin(\alpha) \sqrt{\frac{2}{\rho} P_s} \right) (P_s - P_c)$$

yields $Q_r = 4.77$ gpm. This is smaller than the required flow of 5 gpm. Therefore, a higher P_s is needed. Let $P_s = 1110$ psi for the second trial. The result is $Q_r = 5.28$ gpm. This time Q_r

is higher than 5 gpm. So, let's use a value in between, say 1105 psi, for the third trial. We have $Q_r = 5.025$ gpm at $P_s = 1105$ psi. Since the difference between 5.025 gpm and 5 gpm is less than 1%, one would assume the relief pressure at 5 gpm is very close to 1105 psi.

Example 6-2

Establish a HyPneu circuit to simulate the dynamics of the poppet relief valve described in Example 6-1. The poppet has a weight of 0.068 lbf (0.031 kgf). Use $0.2 \text{ lbf}\cdot\text{s}\cdot\text{in}^{-1}$ ($0.0357 \text{ kgf}\cdot\text{s}\cdot\text{cm}^{-1}$) for damping coefficient in the analysis.

Solution

Figure 6-19 shows a HyPneu circuit that contains the poppet model and the supporting subsystem to simulate the relief valve dynamics. The poppet valve model is constructed based on three key elements:

1. The relation between the action of the mass-damper-spring arrangement (SH5550) and the differential pressure acting on the poppet
2. The flow area model for the poppet (SI6511) that determines the flow area based on the valve displacement determined from SH5550
3. A variable area orifice model (SH5520) which calculates the flow through the poppet in terms of the flow area from SI6511

The following components are used in the simulation:

- Pump: fixed displacement, 10 gpm
(37.854 ℓ /pm) constant flow
- Control valve: 4-way-2-position, pressure drop 200 psid
(13.79 bar) at 10 gpm;
- Control signal: 0 to 0.2 sec direct pump flow to tank; 0.2 sec and above direct flow to poppet valve

Figure 6-20 shows the simulation results along with the experimental data. Note that the steady state pressure is around 1200 psi (82.74 bar). This agrees with the calculation obtained in Example 6-1.

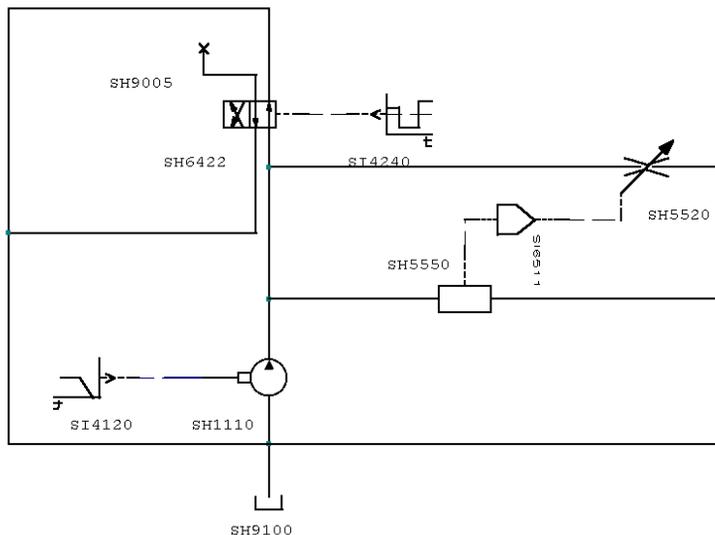


Figure 6-19. HyPneu Circuit of Relief Valve Example 6-2.

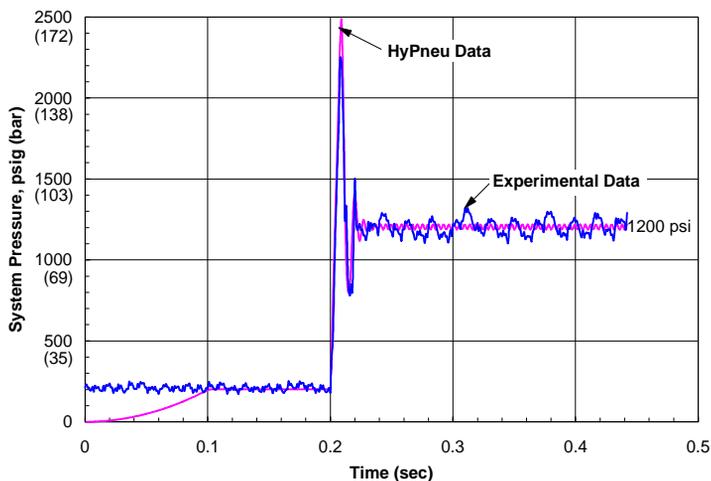


Figure 6-20. HyPneu Data vs. Experimental Data.

Surge Pressure Analysis

Large surge pressures result from the rapid closure of a valve at the end of an active conduit. Since this phenomenon is accompanied by considerable noise, it is often called *water hammer*. Due to the poor response of relief valves, an overshoot of 25 to 50% is usually experienced when the relief valve is called